Report on research visit of Elena Zhizhina
at the Arctic University of Norway, UiT, campus Narvik, in September-October 2018

Recent years there is an increasing interest to non-local integral operators with a kernel of convolution type. Non-local operators and related random jump processes provide a new paradigm of scientific modelling giving a corrective to descriptions based on the second order elliptic and parabolic operators. The relativistic models of quantum mechanics and filed theory, anomalous transport in cell biology, heavy-tailed distributions on the financial markets, or long-range dynamics in geophysical and environmental systems are all beneficiaries of these approachers and techniques. In particular, non-local versions of heat equations describe evolutions with finite speed of propagation, and the zero order convolution type integral operators describe the evolution of key characteristics in various ecological models, in chemical processes, in contact models of population dynamics. One of the interesting example of such problems is the non-local Stefan model that describes the phenomenon of phase transition, for example, between water and ice. In contrast with the local problem, non-local setting develops mushy regions, even if they where not presented initially.

In many applications the environment is not homogeneous, it might have a periodic, locally periodic or random stationary structure. In this case the corresponding integral kernel has a more complicated profile. In addition to a convolution type function the kernel has an additional factor that represents the inhomogeneities of the environment. This structure naturally leads to homogenization problems for non-local operators.

From the mathematical point of view this subject is of great interest because it is at the interface of several fields such as functional analysis, stochastic analysis, probability theory and differential equations.

During my visit at UiT, campus Narvik, I gave several lectures on asymptotic properties of non-local operators. Also, many aspects of this subject have been discussed with my colleagues from UiT. The topics of interest in this area are

- relation between microscopic continuous contact model (a point process in continuum) and an evolution non-local equation for the first correlation function of the model describing the density of the point configuration;
- asymptotics of the density of populations in the continuous non-homogeneous contact model and related structure of the positive spectrum of non-local Schrodinger operators;
- asymptotics of the fundamental solution of non-local heat equations depending on the decay of the corresponding convolution kernel;
- homogenization problems for a non-local operators in a periodic non-homogeneous environment.

During my stay in Narvik we worked in collaboration with Prof. Andrey Piatnitski on the research project "Homogenization of convolution type non-symmetric operators". As a result of this work a new research paper has been written. In the mentioned project we studied the homogenization problems for non-local operators with non-symmetric kernels. We found the effective speed and the effective convolution matrix, and also obtained a number of interesting properties of the corresponding processes, such as the so-called Einstein relation. This relation is of great interest in mathematical physics community.