Report on the visit of Adam Rennie to NTNU (campus Trondheim) in the Fall 2018

My purpose of my visit to Trondheim was to help build a bridge between two fields. My host, Franz Luef, started this process by recognizing that many of the tools of operator algebras had application to the field of time-frequency analysis.

Time frequency analysis is a branch of harmonic analysis which underpins many aspects of our modern communication systems. That tools developed for studying quantum systems provided useful tools was a surprise.

What Franz discovered was that the process of extracting coefficients from a signal and resynthesising the signal defines a representation of an algebra determined by the method of analysis chosen. Then many aspects of assessing the most useful methods of analysis reduce to studying properties of the modules.

The plan was to go beyond what Franz has already achieved, by employing not just operator algebras but the field of non-commutative geometry. The tools of non-commutative geometry were developed over the last three decades to provide a more geometric and computationally tractable approach to the topological invariants provided by operator algebras.

During my visit I gave a mini-course on the foundations of non-commutative geometry, how it is used, and what can be done with it. A primary focus was spectral triples, and I also gave a seminar on related topics. The main attendees were Franz' students and postdocs.

The research we pursued was (initially) related to an open problem in time frequency analysis, related to earlier work by Franz. Essentially, the aim is to show in high dimensions that the natural class of lattices to consider for time-frequency analysis all give rise to suitable "Gabor atoms". This means that the module determined by the analysis and synthesis operations is actually singly generated. This work is still in progress, but seems likely to have a good outcome.

While discussing these problems I also became aware of Franz' work on Heisenberg manifolds, and I believe that there is a lot I can add to this work. In recent years I have looked a lot at "non-commutative dynamical systems" defined using specific kinds of module actions: which arise in the definition of Heisenberg manifolds. Thus another project examining the structure of Heisenberg manifolds has also begun.

Both fields of study, time-frequency analysis and non-commutative geometry, stand to benefit greatly from more interaction. I hope to be part of that interaction for many years to come.