MOTIVIC SYMMETRY

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Roughly speaking algebraic geometry studies the properties of polynomial equations which are encoded in their geometric structures. These geometric structures are inherently of a rigid nature, for example a circle and an ellipse are different geometric objects. Nonetheless these share an important commonality: they divide space into an “inside” and an “outside”. Now imagine a rubber band in the shape of a circle. This rubber band circle is easily stretched into an ellipse or into any number of shapes and back into a circle. All of these shapes share the commonality mentioned above. Homotopy theory is a branch of algebraic topology which studies properties of a “shape” that are preserved even after deforming the shape. Higher dimensional examples quickly become complicated and to handle this complexity these two fields have developed very different and very specialized techniques.

At first glance, because of the dissonance between the viewpoints of rigidity and of deformability, it appears doubtful that homotopy theory has any meaningful role in algebraic geometry. However, the previous two decades or so have borne witness to spectacular applications of the tools from homotopy theory within algebraic geometry. This is based on insights of V. Voevodsky which, following a long line of development initiated by Grothendieck in the 1960s, uncover subtle connections between these subjects. Morel and Voevodsky introduced *motivic homotopy theory*, providing a framework to apply the powerful tools of homotopy theory and algebraic topology to the study of algebraic varieties. This theory was developed to isolate the aspects of geometry of algebraic varieties that influence arithmetic properties, i.e., *motives.* Fundamental algebro-geometric invariants such as algebraic K-theory and motivic cohomology can be studied within this setting. These ideas have proven remarkably successful *viz.* Voevodsky’s celebrated proof of the Milnor conjectures, for which he was awarded the Fields Medal in 2002. Additionally, this work has led to the resolution of other long-standing open problems such as the Beilinson-Lichtenbaum conjectures, the Bloch-Kato conjectures, and the Qullen-Lichtenbaum conjectures.

During my visit I gave three talks. The first two talks detailed work on equivariant homotopy theory. This included joint work with David Gepner on the Adams isomorphism and the tom Dieck splitting theorems in equivariant motivic homotopy theory. The first asserts that there is an equivalence between fixed points and the quotient of a free motivic G-spectrum and the second is a computation of the fixed points of a suspension spectrum. The remaining talk was focused on recent work with Tom Bachmann and Elden Elmanto on power operations on normed motivic ring spectra. In that work, using both equivariant and categorical methods we construct a good theory of power operations for normed algebras over mod-2 motivic cohomology. Some preliminary applications include some splitting results for normed algebras of characteristic two.

I also discussed several ongoing projects with Professor Østvær during the visit. In one, which is also joint with Beaudry, Roendigs, Stojanoska, we consider the homotopy limit problem for Hermitian K-theory. Briefly, there is a motivic version of homotopy fixed points constructed using geometric versions of EG. We show that KQ can be obtained as the motivic homotopy fixed points of the C_2-action on KGL. This converts the usual homotopy limit problem into a comparison of motivic and classical homotopy fixed points of KGL. Solving this problem we find that the KQ is the η-completion of the classical homotopy fixed points of KGL. This recovers and generalizes previous work of Hu-Kriz-Ormsby, Berrick-Karoubi-Schlichting-Ostvaer, Roendigs-Spitzweck-Ostvaer, and Heard.

In the second project, we discussed slice filtrations on equivariant homotopy theory. Previously, we have considered an ad-hoc definition for a slice filtration on motivic C_2-equivariant spectra. As an indication that this filtration is a good one to consider, we have shown that over a field of characteristic zero is the zeroth layer of this filtration is given as the Bredon motivic cohomology spectrum. This is an equivariant version of motivic cohomology; in characteristic zero it can be described via symmetric powers of spheres. During my visit we discussed how to generalize these constructions and results to other groups. Following work of Hill-Yarnell on the topological equivariant slice spectral sequence, we use the motivic constructions of geometric fixed points, obtained in my joint work with Gepner mentioned above. The equivariant filtration is then defined in terms of conditions on the slice connectivity of geometric fixed points.