Visit to NTNU Trondheim, April and June 2019

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There has been a remarkable growth in the theory of stochastic differential equations and their applications in many subject areas, since the pioneering work of Itô and Gihman in the 1940s and 1950s. In their applications, stochastic differential equations are used to describe the evolution of quantities driven by random phenomena. Typical examples include the future evolution of financial variables such as stock prices or interest rates, or the movement of physical objects affected by random forces. More often than not, stochastic differential equations have no known solution as a given function of the driving stochastic processes. Their efficient simulation while retaining any key characteristics of the solution is thus an important task in applications. A key characteristic, that is important in a wide spectrum of applications, is when the solution evolves on a manifold. An example is that of a satellite whose movement around its centre of gravity is disturbed by random forces. A point on the surface of the satellite has a constant distance to the satellite's centre of gravity, and the stochastic differential equation describing this movement evolves therefore on a sphere. Solving the SDE numerically to track the movement enables one to take action when the movement becomes too erratic. As the true stochastic process describing this movement evolves on a manifold, the sphere, it is desirable for the numerical solution tracking this movement to do so also. This feature can prove to be pivotal also for the stability of an approximation; there are cases where a standard Taylor-based numerical approximation becomes unstable and explodes, while the one evolving on the manifold does not. The flow map describes the transport of the initial condition to the solution of the stochastic differential equation at a future time. Its log plays a crucial role in the context of approximating a solution that evolves on a manifold. In joint work with Ebrahimi-Fard, Malham and Patras, we show that it is a Lie series for systems driven by continuous processes (semimartingales). This is an important property when developing stochastic Lie group integrators, see for example Malham & Wiese (2009) for the development of such stochastic Lie group integrators for equations driven by Wiener processes.

During the visit, I gave also a series of lectures to MSc students, PhD students and academic staff. In the lectures I gave an introduction to simulation methods in mathematical finance. I presented further joint research work with Simon Malham on the Cox–Ingersoll–Ross (CIR) model. The CIR process is used in many contexts, it is well known as a model for the short rate of interest (Cox, Ingersoll and Ross 1985) and the variance process in the Heston stochastic volatility model (Heston 1993). While many properties of the CIR process are well-understood, no analytic solution in terms of the driving Wiener process is known. Hence efficient computational methods are required in applications. Interestingly, the CIR process behaves very differently depending on its parameters. In one parameter regime, the zero boundary is unattainable, while in the complementary parameter regime, the zero boundary is attainable and at this boundary, the solution is reflected into the positive domain. It is this latter behaviour that makes the numerical treatment of the CIR process notoriously difficult. A recent result by Hefter and Jentzen shows the convergence rate of equidistant time-discretization methods for the CIR model cannot be better than a constant determined by the parameters of the model. This constant can be arbitrarily small in practically relevant cases. In my joint work with Simon Malham we develop an efficient, almost exact simulation method, that does not involve time-discretization. Almost exact here means an order of accuracy of 10^{-10}, though the method is not limited to this accuracy level.

I wish to thank my hosts, Kurusch Ebrahimi-Fard and indeed NTNU, Trondheim, for their hospitality and the Tromsø Research Foundation for their support for this visit.