# Almost Robinson geometry


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An *almost Robinson structure* on an even-dimensional Lorentzian (conformal) manifold is a totally null complex distribution of maximal rank. Such a structure singles out a real null line distribution together with a bundle Hermitian structure on its screen bundle. When the complex distribution is involutive, it is referred to as a *Robinson structure*, a notion introduced by Nurowski and Trautman in 2002. In this case the congruence of curves tangent to the null line distribution is geodesic, and its leaf space acquires a *Cauchy-Riemann (CR) structure*. The history of this topic in fact goes back to Elie Cartan's seminal work on pure spinors. Later, during the Golden Age of general relativity, Robinson structures, under the guise of *non-shearing congruences of null geodesics*, proved fundamental in the study of exact solutions to the Einstein field equations in dimension four. They also provide an elegant geometric articulation of important results of mathematical relativity such as the *Robinson*, *Goldberg-Sachs* and *Kerr theorems*, which were influential in the early formulation of Sir Roger Penrose's *twistor theory*. A recent state-of-the-art account of the subject can be found in the preprints:

- Optical geometries (A. Fino, T. Leistner, A. Taghavi-Chabert), `arXiv:2009.10012`
- Almost Robinson geometries (—), `arXiv:2102.05634`

One of the purposes of my visit to UiT was to give an introductory mini-course on almost Robinson geometry from the perspective of G-structures, drawing on examples from general relativity.

The professorship also gave Prof. Dennis The and myself the opportunity to collaborate on topics of common interest, especially in connection with CR geometry. The starting point of our collaborative research was motivated by my recent article

- Twisting non-shearing congruences of null geodesics, almost CR structures, and Einstein metrics in even dimensions (A. Taghavi-Chabert), *Ann. Mat. Pura Appl. (4)* (2021)

where it is shown that certain classes of Einstein Lorentzian metrics in even dimensions, such as *Taub–NUT–(A)dS metrics*, arise from so-called *(almost) CR–Einstein structures*, that is, a CR analogue of Einstein metrics. On the other hand, Dennis and his collaborators have recently completed a classification of homogeneous CR 5-folds and their complex analogues, the so-called *(complex) integrable Legendrian contact (ILC) structures*, as recorded in

- Homogeneous Levi non-degenerate hypersurfaces in $\mathbf{C}^3$ (B. Doubrov, A. Medvedev, D. The), Math. Z. **297**, 669-709 (2021)
- Homogeneous integrable Legendrian contact structures in dimension five (—), J. Geom. Anal. **30**, 3806-3858 (2020)

A natural question thus arises: which of these homogeneous models admit CR–Einstein structures? Combining our respective expertise, our initial efforts were met with success, yet raised further interesting issues. We anticipate that this collaboration will blossom into a full-fledged mathematical partnership between Dennis and myself.