My visit to the Mathematics Department, University of Bergen

April 19 - May 1, 2022

This is not my first visit to the Department of Mathematics, and hopefully not the last. The outcomes of my long lasting collaboration with Professor Irina Markina have always been an added value for my career. The main purpose of this visit was to continue and further develop our joint work on rolling motions of manifolds, subject to non-holonomic constraints of no-slip and no-twist, away from the limitations of communication online.

Finding methods to solve interpolation problems on Riemannian manifolds in an efficient way, became an important mathematical research objective in recent years. This is due to the positive impact that explicit solutions of such problems may have, mainly in engineering areas such as robotics, computer vision, signal and image processing, and medical imaging technology, just to mention a few.

A good interaction between mathematicians knowledgeable in differential geometry, and engineers working in such areas, can greatly benefit both subjects. Being a mathematician and a researcher at the Institute of Systems and Robotics at the University of Coimbra, an important challenge for me is to establish the grounds that facilitate the interaction with researchers having different backgrounds. Here, at the University of Bergen, I found conditions for a fruitful dialogue and effective collaboration having such objectives in mind.

Many well known approaches for solving interpolation problems on Euclidean spaces have been extended to Riemannian manifolds and are theoretically appealing. However, in most situations one ends up with solutions that are defined implicitly or, even worse, with highly nonlinear differential equations evolving on manifolds, that are extremely hard to solve. Rolling motions of Riemannian manifolds on flat spaces offer a much simpler method to generate interpolating curves, as long as such motions can be described in a manageable form.

During my stay, I worked in strong collaboration with Professor Irina Markina to understand better the relation between two notions of rolling a Riemannian manifold on another and make clear the difference between them, using a language accessible to a wider audience, in particular to researchers having interest in applications. One of those notions, "the extrinsic rolling", makes use of the isometric embedding of the manifolds in an ambient (semi)-

Euclidean vector space V, so that the rollings are described in terms of the action of the group $SE(V)$ of oriented isometries of V. On the other side, "the intrinsic rolling" is a more abstract concept, but a better understanding of one benefits the other. We also concentrated our efforts on rollings of symmetric spaces on flat spaces, and complemented the theoretical results with illustrative examples. This class of Riemannian manifolds is particularly important in robotics, since the configuration of most mechanical systems is described in terms of symmetric spaces, such as spheres and Lie groups. As a consequence, interpolating curves in those spaces are fundamental tools in path planning. Grassmann manifolds, which are also symmetric spaces, are used for face recognition in the context of still imagery or in videos. For instance, a curve that interpolates a set of points in a Grassmann manifold may describe the temporal evolution of an event from which only a short number of scenes was captured.

Stiefel manifolds, which are reductive homogeneous spaces but fail to be symmetric spaces, are also very useful in applications, when in addition to the subspace structure of Grassmann the specific choice of basis vectors is crucial. In our joint work here, we also addressed rolling Stiefel manifolds on flat spaces. Our results will soon be submitted for publication in a mathematical journal.

During my stay I gave a talk, *Interpolation on Riemannian Manifolds*, at the Analysis and PDE Seminar (on Tuesday 26, from 14:15 to 16:00), and also had the chance to interact with postdocs and graduate students.

I would like to thank the University of Bergen for the financial support and for providing excellent working conditions. I am also most grateful to Professor Irina Markina, for many interesting and important discussions, always in a very friendly atmosphere.

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