# My visit of the Department of Mathematics at the University of Bergen, Bergen, Norway.

I am Professor Kenro Furutani, working at Osaka Central Advanced Mathematical Institute, Osaka Metropolitan University, Osaka, Japan. I visited the Department of Mathematics of the University of Bergen, Norway in the period from October 25 to November 22 of 2022. This was my second long visit to the University of Bergen and it was very productive. We started the collaboration with Professor Irina Markina in 2012 when she visited Japan during her sabbatical leave.

Our first project was dedicated to proving the existents of a lattice on some special 2-step nilpotent Lie groups, intimately related to the Clifford algebras, called pseudo H-type Lie groups. The existence of a lattice in this case is closely related to the existence of a basis on the Lie algebra, having rational structure constants. The Lie algebras of our interests appeared in the attempt to study hypo-elliptic operators on some geometric setting that is more general than the vector spaces. One of the most used hypo-elliptic operators got the name sub-Laplacian by the properties that are close to the classical Laplace operator. We proved that pseudo H-type Lie algebras have bases with rational, and even integer, structure constants. Moreover, we gave a method for the construction of such kinds of bases. This result allowed the finding of an infinite series of examples of non-diffeomorphic nil-manifolds, where the sub-Laplacian has a similar spectrum. Since a basis giving the rational structures is not unique, the natural question that arises is the classification of these bases, which will lead to the classification of the corresponding lattices. These will also provide a classification of corresponding nil-manifolds that are quotient spaces of pseudo H-type Lie groups by a lattice. In the previous joint works, we with Professor Markina described the groups of automorphisms of pseudo H-type Lie algebras. During my visit to Bergen in the fall semester of 2022, we started the work on the classification of lattices up to isomorphisms of the lattices. Note that the family of pseudo H-type Lie algebras is infinite and is parametrized by two integer parameters $r$ and $s$. Moreover, for each fixed value of $(r, s)$ one also has a series of algebras, depending on the number $n$ of "irreducible" representations of the Clifford algebra. Note that a classification of lattices is only known for the Heisenberg group that corresponds to the parameters $r = 1$, $s = 0$, and arbitrary positive integer $n$. The series of pseudo H-type Lie algebras $\big((r, s), n\big)$ has some properties of periodicity on parameters t and s. In my visit to Bergen, we concentrated on the basic cases corresponding to the parameters $1 \geq r, s \leq 8$ and $n = 1$. We also restricted ourselves to the most simple bases having the structure constants $\{-1, 0, 1\}$. Even for that restrictive cases, we found typical examples giving non-isomorphic latices. The construction of the bases is related to the behavior of some commutative subgroups in the Clifford groups.

We established several invariants for these subgroups, which are characteristic vectors of integer numbers. The subgroups having equal characteristic vectors allow construction of bases for pseudo H-type Lie algebras, leading to isomorphic lattices on the corresponding subgroups. For the present moment, we are finishing writing down the details for the simplest cases. This project is a long-lasting project that will result in several papers and a continuation of our collaboration.

During the visit, I presented a 2-hours talk "Calabi-Yau structure and Bargmann type transformation on the Cayley projective plane" at the seminar of the group of Analysis and PDE. The main goals of the talk were:

1. To show an explicit expression of *Calabi-Yau structure* on the punctured cotangent bundle $T_0^*(P^2\mathbb{O})$ of the Cayley projective plane $P^2\mathbb{O}$.

2. Based on the above expression, to construct a *Bargmann type transformation* between spaces of holomorphic functions on $T_0^*(P^2\mathbb{O})$ and the $L_2$-space on $P^2\mathbb{O}$.

In the talk, I started to explain the construction of several classical projective spaces based on the matrix representation, which is rather elementary and is useful to understand the exceptional feature of the *Cayley projective plane*. The constructions give the Hopf fibration structures for classical cases and suggest the map to define a Kähler structure on $T_0^*(P^2\mathbb{O})$. Avoiding the technical details I emphasized the geometric nature of defining the Bargmann type transformation.

I had several fruitful conversations during my visit to the Department of Mathematics with postdoc G. Vega Molino on the Cayley projective plane.