Report of the visit to the Mathematics Institute, University of Bergen July – August 2023

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The purpose of the stay was to apprise René Langǿen and Irina Markina about the recent developments in my research on monodromy properties of solutions to Fuchsian differential equations and their confluent limits. During my stay, I conducted discussions sessions on the theory of isomonodromic deformations, as well as relevant topics on conformal field theory.

The version of the Riemann-Hilbert problem which will be relevant to us is the map between the parameters defining a Fuchsian differential equation – and their confluent limits – and the global monodromy properties of its solutions. The theory of isomonodromic deformations has been a valuable tool in attacking these types of problems since the seminal work of the Kyoto group in the early 1980's. As relevant results among many, they proved that the generating function of those deformations – the "tau-function" – possesses the Painlevé property, and that their asymptotic expansions are naturally given in terms of monodromy data. As a later development, complete expansions of the isomonodromic tau-functions were given in what is now called the "Kyiv formula". These definitions establish a procedural solution to the Riemann-Hilbert problem for these families of differential equations.

The complete expansions make use of special functions defined in the representation theory of the Virasoro algebra, called "conformal blocks". Their utility as building blocks of the expansions justify their introduction, also given their relation to the accessory parameters of the differential equations involved. This is particularly useful in the confluent case, where the conformal blocks can be computed far easier than the tau functions.

In the two months of my term in Bergen, I spent half of the time talking about isomonodromic deformations, focusing on the Painlevé VI and Painlevé V systems. The first serves as an easier guideline, given the regular nature of the singularities of the associated linear system. The Painlevé V system is of particular interest to all of us given their relations to the Stokes phenomenon, quadratic differentials and scattering theory of gravitational systems.

During the second half, I discussed elements of conformal field theory which are relevant for the definition of conformal blocks. I focussed on the necessary tools to discuss the regular conformal blocks, appearing in the tau-function of the Painlevé VI transcendent, and the irregular conformal blocks, relevant to the Painlevé V system.

At the end of the stay, we discussed the program of joining the lines of research together, as we intend to map the monodromy parameters in the WKB limit – called "WKB periods" – and the Stokes lines used to calculate the asymptotics of quadratic differentials. The goal would be to get a better grasp of the large-conformal modulus expansion of the Painvelé V tau-function and accessory parameters for Fuchsian differential equations.